

# Moduli spaces of Curves, Integrable Systems and related subjects

## Programme

All talks will take place in Room René Baire, Institut de Mathématique de Bourgogne, floor 4.

	Monday 4	Tuesday 5	Wednesday 6	Thursday 7	Friday 8
9:30 – 10:30		D. Zvonkine	M. Cafasso	N. Orantin	A. Polishchuk
10:30 – 11:00	registration	Break	Break	Break	Break
11:00 – 12:00	D. Yang	J. Guéré	A. Ponso	K. Kozłowski	D. Holmes
12:00 – 14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00 – 15:00	G. Carlet	M. Bolognesi	N. Kitanine	D. Lewanski	A. Chiodo
15:00 – 15:30	Break	Break	Break	Break	Break
15:30 – 16:30	A. du Crest de Villeneuve	G. Borot	S. Abenda	R. Kramer	N. Pagani
16:30 – 17:00	Break	Break	Break	Break	Break
17:00 – 18:00	A. Brini	V. Roubtsov			

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## Book of abstracts

- S. Abenda (U. di Bologna)

TITLE: KP theory, plane-bipartite networks in the disk, and rational degenerations of M-curves

ABSTRACT: We discuss the connection between two objects, naturally arising in the theory of the Kadomtsev-Petviashvili equation: totally nonnegative Grassmannians and rational degenerations of the M-curves (Riemann surfaces with an antiholomorphic involution and the maximal possible number of real ovals) with a collection of marked points. In particular, we show that a KP divisor satisfying the reality and regularity conditions on a degenerate M-curve is canonically associated to any point in the totally non-negative Grassmannian. This research is done in collaboration with P.G. Grinevich (LITP, Moscow)

- J. Guéré (U. de Lyon)

TITLE: From DR hierarchies to tautological classes

ABSTRACT: In 2014, Alexandr Buryak constructed, for each cohomological field theory (CohFT), a new integrable hierarchy based on the geometry of the double ramification cycle and called the DR hierarchy. He then conjectured that it is equivalent to a construction of Dubrovin-Zhang for semi-simple CohFTs. In this talk, we recall the main steps of these two constructions and we will see how their conjectural equivalence leads to conjectural relations in the tautological rings of the moduli spaces of stable curves. It is a joint work with Buryak, Dubrovin, and Rossi.

- A. Brini (Imperial College - London)

TITLE: An ADE triptych

ABSTRACT: I will append a Lie-algebra label (most of the time simply-laced) to a few correspondences relating curve counting invariants of complex algebraic 3-folds, quantum invariants of real 3-manifolds, integrable systems and Frobenius manifolds. The starting point is the generalisation of a famous identification of Chern–Simons theory on the three-sphere with Gromov–Witten theory on an auxiliary Calabi–Yau space (the resolved conifold) to arbitrary three manifolds; I will exhibit a mirror symmetry version of the same correspondence, and its proof for the LMO invariant (joint with Borot), for the case of Clifford-Klein 3-manifolds. This involves a family of spectral curves of relativistic integrable systems, deforming the Lie-algebraic generalisation of the periodic Toda chain. I will mention three independent applications, partly still conjectural, of the construction above, leading to 1-dimensional mirror theorems for Dubrovin-Zhang Frobenius manifolds associated to affine Weyl groups of type ABCDEFG, the quantum cohomology of ADE P1-orbifolds, and the orbifold quantum cohomology of simple surface singularities; in the simply-laced case and at higher genus, the topological recursion and Givental’s quantisation can be used to lift the construction above to provide an ADE version of the Norbury-Scott conjecture, and to reduce the descendent higher genus crepant resolution conjecture for ADE resolutions to genus-zero calculations. Partly for its intrinsic interest, partly for the sake of example, and partly as a pretty lame muscular posture of computing power by the speaker, details of the construction will be provided for the case of E8.

- M. Bolognesi (U. de Montpellier)

TITLE: A variation on a theme of Faber and Fulton

ABSTRACT: In this talk we study the geometry of GIT configurations of  $n$  ordered points on  $\mathbb{P}^1$  both from the birational and the biregular viewpoint. In particular, we prove that any extremal ray of the Mori cone of effective curves of the quotient  $(\mathbb{P}^1)^n // PGL(2)$ , taken with the symmetric polarization, is generated by a one dimensional boundary stratum of the moduli space. Furthermore, we develop some technical machinery that we use to compute its group of automorphisms. (joint work with A.Massarenti)

- G. Borot (MPIM - Bonn)

TITLE: Orbifold-spin Hurwitz numbers and topological recursion

ABSTRACT: I will present recent progress on Zvonkine’s r-ELSV conjecture and topological recursion for r-spin Hurwitz numbers, and their q-orbifold version, based on a joint work with R. Kramer, D. Lewanski, S. Popolitov and S. Shadrin, and earlier works of subsets of these authors. I will also point out that the general result announced by Alexandrov-Chapuy-Eynard-Harnad on topological recursion for certain class of simple and double Hurwitz numbers, when applied to the r-spin case in a naive way (as it is a situation not respecting their assumptions) does not predict the correct spectral curve.

- M. Cafasso (U. d'Angers)

TITLE: Noncommutative Painlevé equations and systems of Calogero type.

ABSTRACT: All Painlevé equations can be written as the motion of a particle under a time-dependent potential, and as such they admit a natural generalisation to the case of several particles with an interaction of Calogero type (rational, trigonometric or elliptic). In this talk, I will show that these many-particles Hamiltonian systems admit an isomonodromic formulation, thus answering to a question raised by Takasaki. The isomonodromic formulation can be used, in combination with discrete Schlesinger transforms, to produce solutions; time permitting I will illustrate the method for the case of the second Painlevé equation. This is a joint work with Marco Bertola and Vladimir Roubtsov.

- G. Carlet (U. de Bourgogne)

TITLE: Normal forms of multidimensional Dubrovin-Novikov Poisson structures.

ABSTRACT: First we will review the theory of classification of dispersive Poisson (and bi-Hamiltonian structures) under Miura type transformations. Then we will present our recent results on the cohomology of scalar multidimensional Poisson structures of DN type and the complete classification of their dispersive deformations in the case of two independent variables.

- A. Chiodo (IMJ - Paris 6)

TITLE: Néron models and genus-one double ramification cycles via Picard functors

ABSTRACT: Néron models of Jacobians are naturally described via Picard functors. Over a discrete valuation ring, this can be obtained by Raynaud's theorem via a quotient of the non-separated Picard functor. We can also present a direct approach within the separated functor  $Pic^0$  of twisted curves. Recently Holmes extended Raynaud's approach on a base scheme of dimension greater than one and was able to provide in this way a universal Néron model over moduli of curves. This construction admits several applications (e.g. the study of limit linear series by Biesel and Holmes). It also allows a new definition of the Double Ramification locus (DR) parametrizing curves equipped with a principal divisor. In collaboration with Holmes, we compute this cycle in genus one and match the formula of Janda-Pandharipande-Pixton-Zvonkine.

- A. du Crest de Villeneuve (U. d'Angers)

TITLE: Polynomial Tau Functions and Bilinearization of the Drinfeld–Sokolov Hierarchies

ABSTRACT: The Drinfeld–Sokolov hierarchies are sequences of partial differential equations introduced in 1985. Each sequence of PDE's is called an integrable hierarchy and possesses a bihamiltonian structure. Moreover, one can express all the components of any solution as logarithmic derivatives of a single function called the tau function. In this talk I aim to show how to compute polynomial tau functions (the simplest ones) of the Drinfeld–Sokolov hierarchies in terms of Toeplitz determinants. Furthermore, this allows us to search for Hirota equations that could be satisfied by the computed tau functions. For instance, in the Drinfeld–Sokolov hierarchy of type B2, we showed that there is no Hirota equation of degree 2 nor 4 satisfied by the computed tau functions and only one for degree 6 and 8.

- D. Holmes (U. of Leiden)

TITLE: A universal resolution of the Abel-Jacobi map

ABSTRACT: It is well-known that the Abel-Jacobi map does not extend over the boundary of the moduli space of stable marked curves. We consider the problem of resolving this map by making blowups of the moduli space, with a particular focus on extending the double ramification cycle. We construct a certain 'universal resolution' of the Abel-Jacobi map, and show that the resulting extension of the double ramification cycle coincides with that constructed by Graber and Vakil via virtual fundamental classes.

- N. Kitanine (U. de Bourgogne)

TITLE: Quantum separation of variables and scalar products

ABSTRACT: Determinant representations for scalar products of so-called off-shell and on-shell states play a crucial role in the theory of quantum integrable models as they open a way to compute and analyse the correlation functions and form factors. In this presentation I'll give a brief description of a systematic way to construct such representations in the framework of the quantum separation of variables approach.

- K. Kozłowski (ENS Lyon)

TITLE: On singularities of dynamic response functions in the massless regime of the XXZ chain

ABSTRACT: I will explain how, starting from the large-volume behaviour of the form factors of local operators, one can construct the thermodynamic limit of the massless form factor expansion of dynamical two-point functions in the XXZ chain. The massless form factor expansion provides one with an efficient tool allowing one to fully describe the singular behaviour of the dynamic response functions - *i.e.* the space and time Fourier transforms of two-point functions. The characterisation of the singular structure of the response function obtained in this work builds on a first principle based analysis carried directly at the microscopic level. On top of being free from any hypothetical correspondence with a field theory, the analysis unravels a new class of edge exponents stemming from collective excitations, which was not accounted for by the existing heuristic approaches. Finally, the analysis provides a very simple picture allowing one to reduce the manifestation of universal features characteristic of the Luttinger Liquid universality class to the presence of certain singularities in the form factors of local operators and to consequences of a classical asymptotic analysis of multiple integrals.

- R. Kramer (U. van Amsterdam)

TITLE: Deformations of Poisson pencils of hydrodynamic type via bi-Hamiltonian cohomology

ABSTRACT: Many well-known integrable hierarchies can be expressed in terms of Poisson pencils of hydrodynamic type in several dependent variables depending on an independent variable. These pencils have a natural action of the Miura group defined on them, and hence, it is a natural problem to classify their orbits. In 2004, Dubrovin-Liu-Zhang defined so-called central invariants, and showed that pencils with different central invariants are inequivalent under this action. They conjectured that conversely, any central invariant corresponds to a pencil, a statement proved by Carlet-Posthuma-Shadrin using bi-Hamiltonian cohomology. In this talk, I will review this and give a new proof of Dubrovin-Liu-Zhang's theorem, using similar techniques. Based on joint work with Guido Carlet and Sergey Shadrin.

- D. Lewanski (U. van Amsterdam)

TITLE: Tautological ring and PPZ relations.

ABSTRACT: We use relations derived by Pandharipande, Pixton and Zvonkine to obtain some restrictions on the dimensions of the tautological rings of the open moduli spaces of curves. From a joint work with Labib, Kramer and Shadrin.

- N. Orantin (EPFL - Lausanne)

TITLE: Topological recursion and quantization

ABSTRACT: In this talk, I will review a new formalism proposed by Kontsevich and Soibelman interpreting the topological recursion formalism as the result of the quantization of some quadratic Lagrangian in a symplectic space. I will present some examples as well as the link with Givental's point of view on Cohomological Field Theories. This talk is based on a joint work with Andersen, Borot and Chekhov.

- A. Polishchuk (U. of Oregon)

TITLE: Moduli spaces of curves with non-special divisors.

ABSTRACT: In this talk I will discuss the moduli spaces of pointed curves with possibly non-nodal singularities such that the marked points form a nonspecial ample divisor. I will show that such curves have natural projective embeddings, with a canonical choice of homogeneous coordinates up to rescaling. Using Groebner bases technique this leads to the identification of the moduli with the quotient of an affine scheme by the torus action. These moduli spaces also have a natural interpretation in terms of the Krichever map. In the genus 1 case I will describe explicitly the corresponding GIT stability conditions.

- N. Papanicolaou (U. of Liverpool)

TITLE: The indeterminacy of the Abel-Jacobi maps

ABSTRACT: The Abel-Jacobi morphisms are the sections of the forgetful morphism from the universal Jacobian to the corresponding moduli space of smooth pointed curves. When the source and target moduli spaces are compactified, these morphisms can be reinterpreted as rational maps, and it is natural to ask for their locus of indeterminacy. We explicitly characterize the indeterminacy locus, which depends on the chosen compactification of the universal Jacobian, when the source is compactified to the Deligne-Mumford moduli space of stable curves  $\bar{M}_{g,n}$ . We show that there exists always a compactification such that the given Abel-Jacobi map extends to a well-defined morphism on  $\bar{M}_{g,n}$ . This offers an approach to construct and compute the class of several different extensions of the double ramification cycle, one for each compactification of the universal Jacobian. This is a joint work with Jesse Kass.

- A. Ponno (U. di Padova)

TITLE: The quantum KdV equation in condensed matter physics

ABSTRACT: A perturbative deduction of the quantum Korteweg - de Vries equation (qKdV) is presented. More precisely, it is shown that the dynamics of low temperature phonons in large size, one dimensional arrays of atoms is described, to leading order in Hamiltonian perturbation theory, by the standard qKdV equation. The result holds for any smooth pairwise interatomic potential. The integrability properties of the qKdV equation are thus relevant to the dynamics of the array at thermodynamic equilibrium.

- V. Roubtsov (U. d'Angers)

- D. Yang (MPIM - Bonn)

TITLE: Cubic Hodge integrals and random matrices

ABSTRACT: We prove the Hodge–GUE correspondence conjecture on an explicit relationship between special cubic Hodge integrals over the moduli space of stable algebraic curves and enumeration of ribbon graphs with even valencies. We also discuss the conjectural relationship between the cubic-Hodge integrals satisfying the local Calabi–Yau condition and the Bogoyavlensky–Toda hierarchy (aka Fractional KdV). The talk is based on a series of joint works with B. Dubrovin, S.-Q. Liu and Y. Zhang.

- D. Zvonkine (IMJ - Paris 6)

TITLE: Plugging  $r = 0$  into the space of  $r$ -th roots.

ABSTRACT: Consider a complex curve  $C$  endowed with line bundle  $L$  whose  $r$ -th tensor power is the trivial or the canonical line bundle. The moduli space of pairs  $(C, L)$  is a ramified covering of the moduli space  $\overline{\mathcal{M}}_{g,n}$  of algebraic curves. It carries several natural cohomology classes whose projections to  $\overline{\mathcal{M}}_{g,n}$  turn out to be polynomial in  $r$ . We will state several theorems and conjectures that relate the constant term of this polynomial (obtained by plugging  $r = 0$ ) to the Poincaré dual cohomology classes of important geometric loci in  $\overline{\mathcal{M}}_{g,n}$ . This is joint work with F. Janda, R. Pandharipande, and A. Pixton.